

Analysis of the difference between the quarterly seasonally adjusted series and the summed monthly seasonally adjusted series

Consider a quarterly time series $Q_{actual}(t)$, where each quarter's value is the sum of 3 monthly values from the series $M_{actual,i}(t)$, with each following a simple multiplicative model

$$M_{actual,i}(t) = T_i(t)(1+S_i(t))(1+I_i(t)) \quad (i=1,2,3)$$

where $T_i(t)$, $S_i(t)$ and $I_i(t)$ denote the trend, seasonal and irregular components respectively with the standard properties. (Note that this definition of a multiplicative model varies slightly from the X-11 form where $1+S_i(t)$ and $1+I_i(t)$ comprise the respective seasonal and irregular components.)

Then

$$Q_{actual}(t) = T(t)(1+S(t))(1+I(t))$$

where

$$T(t) = \sum_{i=1}^3 T_i(t), \quad S(t) = \sum_{i=1}^3 u_i(t)S_i(t), \quad I(t) = \sum_{i=1}^3 v_i(t)I_i(t)$$

and the weights $u_i(t)$, $v_i(t)$ satisfy

$$u_i(t) = \frac{T_i(t)}{T(t)}, \quad v_i(t) = u_i(t) \frac{1+S_i(t)}{1+S(t)}, \quad \sum_{i=1}^3 u_i(t) = \sum_{i=1}^3 v_i(t) = 1.$$

Note that the $u_i(t)$ are non-seasonal, but the $v_i(t)$ have seasonal character.

The sum of the seasonally adjusted monthly series is the aggregate

$$Q_{agg}(t) = \sum_{i=1}^3 T_i(t)(1+I_i(t)) = T(t) + T(t) \sum_{i=1}^3 u_i(t)I_i(t)$$

whereas the direct seasonally adjusted quarterly series is

$$Q_{dir}(t) = T(t)(1+I(t)) = T(t) + T(t) \sum_{i=1}^3 v_i(t)I_i(t).$$

In particular

$$Q_{dir}(t) = Q_{agg}(t) + T(t) \sum_{i=1}^3 u_i(t) \frac{S_i(t) - S(t)}{1+S(t)} I_i(t)$$

The above analysis shows that $Q_{agg}(t)$ and $Q_{dir}(t)$ are different, except in the unusual case where the three monthly seasonal factors are the same. Both the direct quarter and aggregated monthly adjustments yield irregulars that are proportional to the trend $T(t)$. However, unlike the aggregated monthly adjustments, $Q_{agg}(t)$, the directly adjusted quarterly series, $Q_{dir}(t)$, and the difference

$Q_{dir}(t)$ - $Q_{agg}(t)$, have irregulars whose variances typically have seasonal character (second order seasonality). On these grounds $Q_{agg}(t)$ is the appropriate seasonal adjustment.

However the situation becomes more complicated when we take into account the fact that we can, at best, only obtain estimates of the various components concerned. If the $S_i(t)$ and $S(t)$ are estimated by $\hat{S}_i(t)$ and $\hat{S}(t)$ then the summed monthly and directly adjusted quarterly series become

$$\hat{Q}_{agg}(t) = Q_{agg}(t) + T(t) \sum_{i=1}^3 u_i(t) \frac{S_i(t) - \hat{S}_i(t)}{1 + \hat{S}_i(t)}$$

$$\hat{Q}_{dir}(t) = Q_{dir}(t) + T(t) \frac{S(t) - \hat{S}(t)}{1 + \hat{S}(t)}$$

where the second terms of the above expressions are seasonal and reflect the proportionate errors in estimating $I+S_i(t)$ and $I+S(t)$ respectively. Even when one takes into account the seasonal structure present in the irregular component of $Q_{dir}(t)$, it may well be true that it is more difficult to extract the seasonal patterns from monthly series than the quarterly, resulting in a quarter adjustment with better properties than the summed monthly series.